

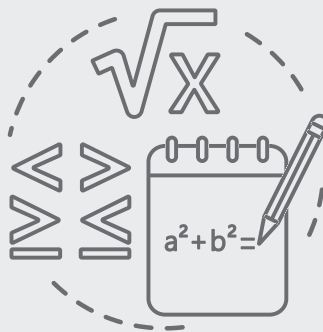
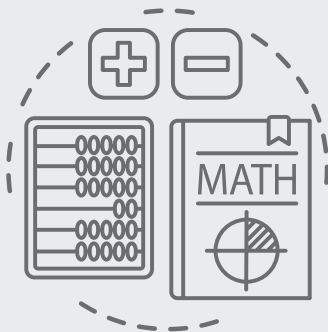


**THE QUESTION**

# How Can Teachers Help Students Who Lack Foundational Math Skills But Have Been Passed Into Advanced Math Classes?

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Under the best of circumstances, the students in any class have a wide variety of strengths and weaknesses, so teaching can seem like a balancing act. At times, due to intentional or unintentional policies, we find a substantial percentage of students lacking what we would consider foundational skills.





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## THE EVIDENCE

### What Doesn't Work

The “obvious” solutions don’t always work: for example, “doubling up” on math prep can sometimes have advantages and sometimes improve test scores but can also increase failure rates (Nomi & Allensworth 2013, U.S. Department of Education 2018). Likewise, within-class differentiation can be problematic. In English Language Arts, for example, separating a class into reading groups where the skilled young readers discuss plot and motive while remedial readers get help sounding out words *increases* differences in student performance. (After all, the remedial readers didn’t get to engage with meaningful content!) A focus on “missing” skills in mathematics presumes a rigidly hierarchical curriculum where you have to know A before you can do B, before you do C, etc. It doesn’t have to be that way: it’s often possible for students to learn core ideas in the context of problem solving rather than assuming that they need the core understandings before proceeding. For one example of how a school opened up curricular practice, see Horn (2007).

### Worthy Tasks

One of the very useful things to come out of an equity-oriented approach known as complex instruction (see Cohen & Lotan 1997 or better, follow the leads in Google) is the concept of “group worthy math problems”—tasks that have multiple entry points and allow for multiple solutions. A well-framed task might be solvable in a “brute force” way, through the clever use of a technique just learned, and possibly (though it may not have been covered in the past week) through graphing or algebra. Different students might approach the task in different ways, so more students have pathways into the core content. Then, comparing and contrasting solutions and showing how they connect is something that all students profit from. Those with less sophisticated approaches see how their ideas link to other methods, including recent content; those who used, say, an algebra solution, may see interesting things when they examine a graphical solution. This kind of problem enables all students to engage with the core mathematics, rather than segregating out those who lack certain skills.

Here’s an unexciting-looking but very rich example. Consider the following task, which is discussed in some detail in Schoenfeld 2019:

Train A leaves a train station at noon and travels at a steady speed of 50 miles per hour. Three hours later Train B leaves the station on a parallel track, traveling at a steady speed of 60 miles per hour. How long does it take for Train B catch up with Train A?

This is supposedly a ten-minute exercise in an algebra class. I’ve asked college students to do this problem in the way that feels most comfortable and to write their solutions on chart paper, which I post. Many students make tables, stretching to either 15 or 18 hours. (There is some question as to when you start.) Some draw graphs; there’s the same question as to whether the lines representing the two trains intersect at  $t = 15$  or  $t = 18$ . Finally, there’s the slick algebra solution. As we proceed through the solutions, those who used tables get to see the information they used in the graphs and see how it’s condensed in the algebra. Then I ask a question for all students: Can you see train B catching up to train A in your representation? This question is most challenging for those who used algebra. If you view math as being about sense making, some of the hierarchical structure crumbles—and making the connections is good for all students.



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## CONCLUSION

### Teaching for Robust Understanding

A key to making all of this work is establishing discourse structures in class that involve students being active participants in sense making. Doing so involves a shift in perspective, from “What should teachers do” to “How are students experiencing instruction, and what kind of sense making are they doing?” That shift lies at the heart of the Teaching for Robust Understanding (TRU) Framework (2018), which you can find at <https://truframework.org/>. Take a look at the TRU tools, specifically the Observation Guide at <https://truframework.org/tru-observation-guide/> and the Conversation guide at <https://truframework.org/tru-conversation-guide/>. They point to things you can look for and questions you can ask yourself when you plan and review lessons. For example, the table on the following page lists the “look fors” for content from the observation guide.

## THE MATHEMATICS

*The extent to which central mathematical content and practices, as represented by State or the Common Core State Standards, are present and embodied in instruction. Every student should have opportunities to grapple meaningfully with key ideas and, in doing so, to become a knowledgeable, flexible and resourceful mathematical thinker and problem solver. Teachers should have opportunities to consider and discuss how each lesson's activities connect to the concepts, practices, and habits of mind they want students to develop over time.*

### Each Student...

- Engages with grade level mathematics in ways that highlight important concepts, procedures, problem solving strategies and applications
- Has opportunities to develop productive mathematical habits of mind
- Has opportunities for mathematical reasoning, orally and in writing, using appropriate mathematical language
- Explains their reasoning processes as well as their answers

### Teachers...

- Highlight important ideas and provide opportunities for students to engage with them
- Use materials or assignments that center on key ideas, connections and applications
- Explicitly connect the lesson's big ideas to what has come before and will be done in the future
- Support the purposeful use of academic language and of representations (e.g., graphs, tables, symbols) central to mathematics
- Support students in seeing mathematics as being coherent, connected and comprehensible

The example discussed above shows that even mundane curricular tasks (and there are lots in our curricula!), when opened up, invite many more students into mathematically productive dialogues. If you try to make things like these happen, then you'll be on the way to having a class in which both students who need to build foundational skills and those who are fluent in the most recently studied techniques can interact profitably.

### References

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